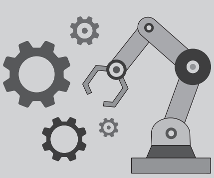
30 *Years*

Previous Solved Papers

GATE 2024

Instrumentation Engineering

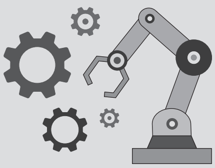


- Fully solved with explanations
- Analysis of previous papers
- Topicwise presentation
- Thoroughly revised & updated

MADE EASY
PUBLICATIONS

www.madeeasy.in

MADE EASY Publications Pvt. Ltd.
Corporate Office: 44-A/4, Kalu Sarai (Near Hauz Khas Metro Station), New Delhi-110016 | **Ph. :** 9021300500
Email : infomep@madeeasy.in | **Web :** www.madeeasypublications.org



GATE - 2024
Instrumentation Engineering
Topicwise Previous GATE Solved Papers (1994-2023)

Editions

1^{st} Edition	:	2011
2^{nd} Edition	:	2012
3^{rd} Edition	:	2013
4^{th} Edition	:	2014
5^{th} Edition	:	2015
6^{th} Edition	:	2016
7^{th} Edition	:	2017
8^{th} Edition	:	2018
9^{th} Edition	:	2019
10^{th} Edition	:	2020
11^{th} Edition	:	2021
12^{th} Edition	:	2022
13^{th} Edition	**:**	**2023**

Preface


B. Singh (Ex. IES)

Over the period of time the GATE examination has become more challenging due to increasing number of candidates. Though every candidate has ability to succeed but competitive environment, in-depth knowledge, quality guidance and good source of study is required to achieve high level goals.

The new edition of **GATE 2024 Solved Papers : Instrumentation Engineering** has been fully revised, updated and edited. The whole book has been divided into topicwise sections.

At the beginning of each subject, analysis of previous papers are given to improve the understanding of subject.

I have true desire to serve student community by way of providing good source of study and quality guidance. I hope this book will be proved an important tool to succeed in GATE examination. Any suggestions from the readers for the improvement of this book are most welcome.

B. Singh (Ex. IES)
Chairman and Managing Director
MADE EASY Group

GATE-2024

Instrumentation Engineering

CONTENTS

Engineering Mathematics

UNIT

I

CONTENTS

Engineering Mathematics

Syllabus

Linear Algebra : Matrix algebra, systems of linear equations, consistency and rank, Eigen value and Eigen vectors.

Calculus : Mean value theorems, theorems of integral calculus, partial derivatives, maxima and minima, multiple integrals, Fourier series, vector identities, line, surface and volume integrals, Stokes, Gauss and Green's theorems.

Differential equations : First order equation (linear and nonlinear), second order linear differential equations with constant coefficients, method of variation of parameters, Cauchy's and Euler's equations, initial and boundary value problems, solution of partial differential equations: variable separable method.

Analysis of complex variables : Analytic functions, Cauchy's integral theorem and integral formula, Taylor's and Laurent's series, residue theorem, solution of integrals.

Probability and Statistics : Sampling theorems, conditional probability, mean, median, mode, standard deviation and variance; random variables: discrete and continuous distributions: normal, Poisson and binomial distributions.

Numerical Methods : Matrix inversion, solutions of non-linear algebraic equations, iterative methods for solving differential equations, numerical integration, regression and correlation analysis.

Analysis of Previous GATE Papers

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
1994	–	–	–
1995	–	–	–
1996	–	–	–
1997	–	–	–
1998	1	–	**1**
1999	–	–	–
2000	–	–	–
2001	–	–	–
2002	–	1	**2**
2003	–	1	**2**
2004	–	–	–
2005	–	2	**4**
2006	1	4	**9**
2007	2	4	**10**
2008	2	7	**16**

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2009	–	2	**4**
2010	2	2	**6**
2011	2	3	**8**
2012	4	6	**16**
2013	3	5	**13**
2014	3	4	**11**
2015	3	5	**13**
2016	3	3	**9**
2017	4	3	**10**
2018	4	4	**12**
2019	4	3	**10**
2020	5	5	**15**
2021	5	3	**11**
2022	4	4	**12**
2023	4	4	**12**

1 Linear Algebra

1.1 The rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix}$ is

(a) 0 (b) 1
(c) 2 (d) 3

[2000 : 1 M]

1.2 The necessary condition to diagonalize a matrix is that
(a) its all eigen values should be distinct
(b) its eigen vectors should be independent
(c) its eigen value should be real
(d) the matrix is non-singular

[2000 : 1 M]

1.3 A system of equations represented by $AX = 0$, where X is a column vector of unknowns and A is matrix containing coefficients, has a nontrivial solution when A is
(a) nonsingular
(b) singular
(c) symmetric
(d) Hermetian

[2003 : 1 M]

1.4 Let A be a 3 × 3 matrix with rank 2. Then $AX = 0$ has
(a) only the trival solution $X = 0$
(b) one independent solution
(c) two independent solutions
(d) three independent solutions

[2005 : 1 M]

1.5 Identify which one of the following is an eigenvector of the matrix $A = \begin{bmatrix} 1 & 0 \\ -1 & -2 \end{bmatrix}$

(a) $[-1 \quad 1]^T$ (b) $[3 \quad -1]^T$
(c) $[1 \quad -1]^T$ (d) $[-2 \quad 1]^T$

[2005 : 1 M]

1.6 For a given 2 × 2 matrix A, it is observed that,

$$A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = -\begin{bmatrix} 1 \\ -1 \end{bmatrix} \text{ and } A\begin{bmatrix} 1 \\ -2 \end{bmatrix} = -2\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Then matrix A is

(a) $A = \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$

(d) $A = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}$

[2006 : 2 M]

1.7 Let A be an $n \times n$ real matrix such that $A^2 = I$ and y be an n-dimensional vector.
Then the linear system of equations $Ax = y$ has
(a) no solution
(b) a unique solution
(c) more than one but finitely many independent solutions
(d) infinitely many independent solutions

[2007 : 1 M]

1.8 Let $A = [a_{ij}]$, $1 \le i, j \le n$, with $n \ge 3$ and $a_{ij} = i.j$.
Then the rank of A is
(a) 0 (b) 1
(c) $n - 1$ (d) n

[2007 : 2 M]

1.9 Let $P \ne 0$ be a 3 × 3 real matrix. There exist linearly independent vectors x and y such that $PX = 0$ and $PY = 0$. The dimension to the range space of P is
(a) 0 (b) 1
(c) 2 (d) 3

[2009 : 1 M]

1.10 The eigen values of a (2 × 2) matrix X are –2 and –3. The eigen values of the matrix $(X + I)(X + 5I)$ are
(a) –3, –4 (b) –1, –2
(c) –1, –3 (d) –2, –4

[2009 : 2 M]

1.11 The matrix $P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ rotates a vector about the axis $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ by angle of

(a) 30°
(b) 60°
(c) 90°
(d) 120°

[2009 : 2 M]

1.12 A real $n \times n$ matrix $A = \{a_{ij}\}$ is defined as follows: $a_{ij} = i$, if $i = j$, otherwise 0
The summation of all n eigen values of A is

(a) $n(n+1)/2$
(b) $n(n-1)/2$
(c) $\dfrac{n(n+1)(2n+1)}{6}$
(d) n^2

[2010 : 1 M]

1.13 X and Y are non-zero square matrices of size $n \times n$. If $XY = 0_{n \times n}$ then

(a) $|X| = 0$ and $|Y| \neq 0$
(b) $|X| \neq 0$ and $|Y| = 0$
(c) $|X| = 0$ and $|Y| = 0$
(d) $|X| \neq 0$ and $|Y| \neq 0$

[2010 : 2 M]

1.14 The matrix $M = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ has eigen values –3, –3, 5. An eigen vector corresponding to the eigen value 5 is $[1 \;\; 2 \;\; -1]^T$. One of the eigen vectors of the matrix M^3 is

(a) $[1 \;\; 8 \;\; -1]^T$
(b) $[1 \;\; 2 \;\; -1]^T$
(c) $\left[1 \;\; \sqrt[3]{2} \;\; -1\right]^T$
(d) $[1 \;\; 1 \;\; -1]^T$

[2011 : 1 M]

1.15 The series $\sum_{m=0}^{\infty} \frac{1}{4^m} (x-1)^{2m}$ converges for

(a) $-2 < X < 2$
(b) $-1 < X < 3$
(c) $-3 < X < 1$
(d) $X < 3$

[2011 : 2 M]

1.16 Given that: $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the value A^3 is

(a) $15A + 12I$
(b) $19A + 30I$
(c) $17A + 15I$
(d) $17A + 21I$

[2012 : 2 M]

1.17 The dimension of the null space of the matrix $\begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$ is

(a) 0
(b) 1
(c) 2
(d) 3

[2013 : 1 M]

1.18 One pair of eigen vectors corresponding to the two eigen values of the matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 \\ -j \end{bmatrix}, \begin{bmatrix} j \\ -1 \end{bmatrix}$
(b) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}$
(c) $\begin{bmatrix} 1 \\ j \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
(d) $\begin{bmatrix} 1 \\ j \end{bmatrix}, \begin{bmatrix} j \\ 1 \end{bmatrix}$

[2013 : 2 M]

1.19 Given:

$x(t) = 3\sin(1000\pi t)$ and $y(t) = 5\cos\left(1000\pi t + \frac{\pi}{4}\right)$

The X-Y plot will be

(a) a circle
(b) a multi-loop closed curve
(c) a hyperbola
(d) an ellipse

[2014 : 1 M]

1.20 A scalar valued function is defined as $f(\mathbf{X}) = \mathbf{X}^T \mathbf{A} \mathbf{X} + \mathbf{b}^T \mathbf{X} + c$, where $\mathbf{A}$ is a symmetric positive definite matrix with dimension $n \times n$; $\mathbf{b}$ and $\boldsymbol{x}$ are vectors of dimension $n \times 1$. The minimum value of $f(\mathbf{X})$ will occur when $\mathbf{X}$ equals

(a) $(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{b}$
(b) $-(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{b}$
(c) $-\left(\dfrac{\mathbf{A}^{-1}\mathbf{b}}{2}\right)$
(d) $\dfrac{\mathbf{A}^{-1}\mathbf{b}}{2}$

[2014 : 2 M]

1.21 For the matrix $\mathbf{A}$ satisfying the equation given below, the eigen values are

$$[A]\begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

(a) $(1, -j, j)$ (b) $(1, 1, 0)$
(c) $(1, 1, -1)$ (d) $(1, 0, 0)$

[2014 : 2 M]

1.22 Let A be an $n \times n$ matrix with rank $r(0 < r < n)$. Then $AX = 0$ has p independent solutions, where p is
(a) r (b) n
(c) $n - r$ (d) $n + r$

[2015 : 1 M]

1.23 A straight line of the form $y = mx + c$ passes through the origin and the point $(x, y) = (2, 6)$. The value of m is________.

[2016 : 1 M]

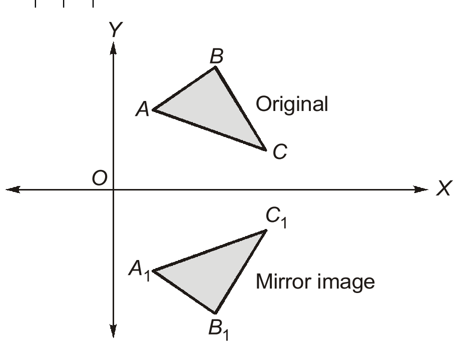
1.24 Consider the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{pmatrix}$ whose eigen values are 1, –1 and 3. Then Trace of $(A^3 - 3A^2)$ is_____.

[2016 : 2 M]

1.25 The eigen values of the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ 0 & 5 & 6 \\ 0 & -6 & 5 \end{bmatrix}$ are
(a) $-1, 5, 6$ (b) $1, -5 \pm j6$
(c) $1, 5 \pm j6$ (d) $1, 5, 5$

[2017 : 1 M]

1.26 The figure shows a shape ABC and its mirror image $A_1 B_1 C_1$ across the horizontal axis (X-axis). The coordinate transformation matrix that maps ABC to $A_1B_1C_1$ is



(a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

[2017 : 1 M]

1.27 If $\boldsymbol{v}$ is a non-zero vector of dimension 3×1, then the matrix $A = \boldsymbol{v}\boldsymbol{v}^T$ has rank = ________

[2017 : 1 M]

1.28 Let N be a 3 by 3 matrix with real number entries. The matrix N is such that $N^2 = 0$. The eigen values of N are
(a) 0, 0, 0 (b) 0, 0, 1
(c) 0, 1, 1 (d) 1, 1, 1

[2018 : 1 M]

1.29 Consider two functions $f(x) = (x - 2)^2$ and $g(x) = 2x - 1$, where x is real. The smallest value of x for which $f(x) = g(x)$ is ________.

[2018 : 1 M]

1.30 Consider the following system of linear equations:

$$3x + 2ky = -2$$
$$kx + 6y = 2$$

Here, x and y are the unknown and k is a real constant. The value of k for which there are infinite number of solutions is
(a) 3 (b) 1
(c) –3 (d) –6

[2018 : 2 M]

1.31 A 3×3 matrix has eigen values 1, 2 and 5. The determinant of the matrix is________.

[2019 : 1 M]

1.32 The curve $y = f(x)$ is such that the tangent to the curve at every point (x, y) has a Y-axis intercept c, given by $c = -y$. Then $f(x)$ is proportional to
(a) x^{-1} (b) x^2
(c) x^3 (d) x^4 [2019 : 2 M]

1.33 A set of linear equations is given in the form $Ax = b$, where A is a 2×4 matrix with real number entries and $b \neq 0$. Will it be possible to solve for x and obtain a unique solution by multiplying both left and right sides of the equation by A^T (the super script T denotes the transpose and inverting the matrix A^TA? Answer is _____.
(a) Yes, can obtain a unique solution provided the matrix A is will conditioned.
(b) Yes, it is always possible to get a unique solution for any 2×4 matrix A.
(c) Yes, can obtain a unique solution provided the matrix A^TA is well conditional.
(d) No, it is not possible to get a unique solution for any 2×4 matrix A. [2020 : 1 M]

1.34 Consider the matrix $M = \begin{bmatrix} 1 & -1 & 0 \\ 1 & -2 & 1 \\ 0 & -1 & 1 \end{bmatrix}$. One of the eigen vectors of M is

(a) $\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

[2020 : 2 M]

1.35 Given $A = \begin{bmatrix} 2 & 5 \\ 0 & 3 \end{bmatrix}$, the value of the determinant $|A^4 - 5A^3 + 6A^2 + 2I| =$ ______.

[2021 : 2 M]

1.36 Consider the rows vectors $v = [1, 0]$ and $w = [2, 0]$. The rank of the matrix $M = 2v^T v + 3w^T w$, where the superscript T denotes the transpose, is

(a) 3 (b) 2

(c) 4 (d) 1

[2021 : 1 M]

1.37 The determinant of the matrix M shown below is ______.

$$M = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

[2021 : 1 M]

1.38 Given $M = \begin{bmatrix} 2 & 3 & 7 \\ 6 & 4 & 7 \\ 4 & 6 & 14 \end{bmatrix}$, which of the following statement(s) is/are correct?

(a) The rank of M is 2

(b) The rank of M is 3

(c) The rows of M are linearly independent

(d) The determinant of M is 0

[2022 : 1 M]

1.39 The matrix $A = \begin{bmatrix} 4 & 3 \\ 9 & -2 \end{bmatrix}$ has eigenvalues – 5 and 7. The eigenvector(s) is/are ________.

(a) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ -6 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 8 \end{bmatrix}$

[2022 : 2 M]

1.40 Choose solution set S corresponding to the systems of two equations

$$x - 2y + z = 0$$
$$x - z = 0$$

Note: . R denotes the set of real numbers

(a) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \middle| \alpha \in R \right\}$

(b) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \middle| \alpha, \beta \in R \right\}$

(c) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \middle| \alpha, \beta \in R \right\}$

(d) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \middle| \alpha \in R \right\}$

[2023 : 1 M]

1.41 The rank of the matrix A given below is one. The ratio $\frac{\alpha}{\beta}$ is _______ (rounded off to the nearest integer).

$$A = \begin{bmatrix} 1 & 4 \\ -3 & \alpha \\ \beta & 6 \end{bmatrix}$$

[2023 : 2 M]

■■■

Answers Linear Algebra

1.1 (c)	1.2 (a)	1.3 (b)	1.4 (b)	1.5 (b)	1.6 (c)	1.7 (b)
1.8 (b)	1.9 (*)	1.10 (a)	1.11 (*)	1.12 (a)	1.13 (c)	1.14 (b)
1.15 (b)	1.16 (b)	1.17 (b)	1.18 (a, d)	1.19 (d)	1.20 (c)	1.21 (c)
1.22 (c)	1.23 (3)	1.24 (–6)	1.25 (c)	1.26 (d)	1.27 (1)	1.28 (a)
1.29 (1)	1.30 (c)	1.31 (10)	1.32 (b)	1.33 (d)	1.34 (b)	1.35 (4)
1.36 (d)	1.37 (4)	1.38 (a, d)	1.39 (a, c)	1.40 (a)	1.41 (–8)	

Explanations Linear Algebra

1.1 (c)

$$\begin{bmatrix}1 & 2 & 3\\3 & 4 & 5\\4 & 6 & 8\end{bmatrix} R_2-3R_1, R_3-4R_1$$

$$\begin{bmatrix}1 & 2 & 3\\0 & -2 & -4\\0 & -2 & -4\end{bmatrix} R_3-R_2$$

$$\begin{bmatrix}1 & 2 & 3\\0 & -2 & -4\\0 & 0 & 0\end{bmatrix}$$

Number of non-zero rows = 2

$\therefore$ Rank of A = 2

1.2 (a)

By known theorem.
A square matrix is diagonalizable if it has distinct eigen values.

1.3 (b)

$AX = 0$ means system of homogenous equations. Which has only trivial solutions if $|A| \neq 0$ i.e. A is non singular. For non trivial solutions $|A| = 0$ i.e. A must be singular.

1.4 (b)

If r is the rank of matrix A and $n \times n$ is the order of matrix then we shall have $(n - r)$ linearly independent non-trival infinite. Any linear combination of these $(n - r)$ solutions will also be a solution of $AX = 0$.

1.5 (b)

$$A = \begin{bmatrix}1 & 0\\-1 & -2\end{bmatrix}$$

Characteristic equation of A

$$|A-\lambda I| = 0$$

$$\begin{vmatrix}1-\lambda & 0\\-1 & -2-\lambda\end{vmatrix} = 0$$

$$-(1-\lambda)(2+\lambda) = 0$$

$$\Rightarrow \quad \lambda = 1, -2$$

Put $\lambda = 1$ in $[A-\lambda I]\bar{X} = 0$

$$\begin{bmatrix}0 & 0\\-1 & -3\end{bmatrix}\begin{bmatrix}X_1\\X_2\end{bmatrix} = 0$$

$$\Rightarrow \quad X_1 + 3X_2 = 0$$

Solution is $X_2 = -k$

and $\quad X_1 = +3k$

$$\bar{X}_1 = [3 \quad -1]^T$$

Since option (b) in is same ratio of X_1 to X_2

$\therefore$ $[3 \quad -1]^T$ is an eigen vector.

1.6 (c)

Let, 2 × 2 matrix, $A = \begin{bmatrix}a & c\\b & d\end{bmatrix}$

$$A\begin{bmatrix}1\\-1\end{bmatrix} = -\begin{bmatrix}1\\-1\end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix}a & c\\b & d\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}-1\\1\end{bmatrix}$$

$$\Rightarrow \quad a - c = -1 \qquad ...(i)$$

$$b - d = 1 \qquad ...(ii)$$

and $\quad A\begin{bmatrix}1\\-2\end{bmatrix} = -2\begin{bmatrix}1\\-2\end{bmatrix}$

$$\Rightarrow \quad \begin{bmatrix}a & c\\b & d\end{bmatrix}\begin{bmatrix}1\\-2\end{bmatrix} = \begin{bmatrix}-2\\4\end{bmatrix}$$

$$a - 2c = -2 \qquad ...(iii)$$

$$b - 2d = 4 \qquad ...(iv)$$

From equation (i) and (iii)

$$c = 1 \text{ and } a = 0$$

From equation (ii) and (iv)

$$d = -3 \text{ and } b = -2$$

$$\therefore \quad [A] = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

On simplification of option (c)

$$A = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -2 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

1.7 (b)

Given, $A^2 = I$

$$|A^2| = |I|$$

$$|A|\cdot|A| = 1$$

$$|A| = \pm 1$$

So, $|A| \neq 0$, so system of equations $AX = Y$ is consistent, and has unique solution given by $X = A^{-1} Y$.

1.8 (b)

$$A = \begin{bmatrix} 1\cdot 1 & 1\cdot 2 & 1\cdot 3 & \dots & 1\cdot n \\ 2\cdot 1 & 2\cdot 2 & 2\cdot 3 & \dots & 2\cdot n \\ 3\cdot 1 & 3\cdot 2 & 3\cdot 3 & \dots & 3\cdot n \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ n\cdot 1 & n\cdot 2 & n\cdot 3 & \dots & n\cdot n \end{bmatrix}$$

All row are the multiple of first row

$$\therefore \quad A = \begin{bmatrix} 1\cdot 1 & 1\cdot 2 & 1\cdot 3 & \dots & 1\cdot n \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}$$

Rank of $A = 1$.

1.10 (a)

X has eigen values –2, and –3.

So, taking $X = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}$, since additional information is not given, so we can take this value.

$$X + I = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$(X + 5I) = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

$$(X + I)(X + 5I) = \begin{bmatrix} -3 & 0 \\ 0 & -4 \end{bmatrix}$$

So, eigen values are –3 and –4.

Alternate method:

Let, $A = (X + I)(X + 5I) = X^2 + 6X + 5I$ ...(i)

Since, eigen value of $X^2 + 6X + 5I$ is $\lambda^2 + 6\lambda + 5$ where, λ is eigen value of X.

So, substituting values,

$\lambda = -2, \quad \lambda_1 = (-2)^2 - 6 \times 2 + 5 = -3$

$\lambda = -3, \quad \lambda_2 = (-3)^2 - 6 \times 3 + 5 = -4$

So eigen values of

$A = (X + I)(X + 5I)$ are –3 and –4

1.12 (a)

$$a_{i_j} = \begin{cases} i; & i = j \\ 0; & i \neq j \end{cases}$$

$$\Rightarrow \quad [A] = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 2 & 0 & \dots & 0 \\ 0 & 0 & 3 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & n \end{bmatrix}$$

Summation of all n eigen value of A

= Trace of A

= sum of diagonal elements

= 1 + 2 + ... + n

$$= \frac{n}{2}(n+1)$$

1.13 (c)

$$[X]_{n\times n}[Y]_{n\times n} = [0]_{n\times n}$$

$$\det([X][Y]) = \det([Y][X])$$

$$= \det[X]\det[Y]$$

$\therefore \quad \det[X]\det[Y] = 0$

$\Rightarrow \quad \det[X] = 0$ or $\det[Y] = 0$

or both are zero but it is not necessary that one of them is non-zero.

1.14 (b)

If $\lambda_1, \lambda_2, \lambda_3 \dots\dots \lambda_n$ are the eigen values of matrix A, then $\lambda_1^k, \lambda_2^k, \dots \lambda_n^k$ will be the eigen values of A^k.

But the A and A^k will have same eigen vector.

Example:

Let a matrix

$$A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix}$$

eigen values of $A = 2, 7$

eigen values of $A^2 = 4, 49$

eigen vector of matrix A corresponding to eigen value 2 is $[3, \ -2]^T$

eigen vector of matrix A^2 corresponding to eigen value 4 is $[3, \ -2]^T$.

1.15 (b)

$$\sum_{m=0}^{\infty} \frac{1}{4^m}(X-1)^{2m} = \sum_{m=0}^{\infty} \frac{1}{2^{2m}}(x-1)^{2m}$$

$$= \sum_{m=0}^{\infty} \left(\frac{X-1}{2}\right)^{2m}$$

$$= 1+\left(\frac{X-1}{2}\right)^2+\left(\frac{X-1}{2}\right)^4+\ldots..\infty$$

This is a geometric progression it will converges

if $r = \left(\frac{X-1}{2}\right)^2 < 1$

$$\Rightarrow \quad \frac{(X-1)^2}{4} < 1$$

$$\Rightarrow \quad (X-1)^2 < 4$$

$$\Rightarrow \quad -2 < (X-1) < 2$$

$$\Rightarrow \quad -1 < X < 3$$

1.16 (b)

Given, $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$

The characteristic equation is

$$|A - \lambda I| = 0$$

$$\Rightarrow \quad \begin{vmatrix} -5-\lambda & -3 \\ 2 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad \lambda(\lambda+5)+6 = 0$$

$$\Rightarrow \quad \lambda^2+5\lambda+6 = 0$$

$$\Rightarrow \quad \lambda^3+5\lambda^2+6\lambda = 0$$

$$\Rightarrow \quad \lambda^3+5(-5\lambda-6)+6\lambda = 0$$

$$\Rightarrow \quad \lambda^3 = 25\lambda+30-6\lambda = 19\lambda+30$$

$$\therefore \quad A^3 = 19A+30I$$

1.17 (b)

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}_{3\times 3}$$

Order of matrix = 3

Rank = 2

$\therefore$ dimension of null space of $A = 3 - 2 = 1$.

1.18 (a, d)

Eigen values are

$$|A - \lambda I| = 0$$

$$\left|\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right| = 0$$

$$\begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = 0$$

$$\lambda^2 + 1 = 0$$

$$\lambda^2 = -1$$

$$\therefore \quad \lambda = \pm i$$

to find eigen vector,

$$\lambda = +i$$

$$\begin{bmatrix} -i & -1 \\ 1 & -i \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\therefore \quad -iX_1 - X_2 = 0$$

and $\quad X_1 - iX_2 = 0$

clearly,

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -j \end{bmatrix} \text{and} \begin{bmatrix} j \\ 1 \end{bmatrix}, \text{ satisfy}$$

$$\lambda = -i$$

$$\begin{bmatrix} i & -1 \\ 1 & i \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$iX_1 - X_2 = 0 \text{ and } X_1 + iX_2 = 0$$

clearly,

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} j \\ -1 \end{bmatrix} \text{and} \begin{bmatrix} 1 \\ j \end{bmatrix}, \text{ satisfy}$$

Thus, the two eigen value of the given matrix are

$$\begin{bmatrix} 1 \\ -j \end{bmatrix}, \begin{bmatrix} j \\ -1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 \\ j \end{bmatrix}, \begin{bmatrix} j \\ 1 \end{bmatrix}.$$

1.19 (d)

$$x(t) = 3\sin(1000\pi t)$$

$$y(t) = 5\cos\left(1000\pi t + \frac{\pi}{4}\right)$$

at $t = 0$,

$$x(0) = 0$$

$$y(0) = 5\cos\frac{\pi}{4} = \frac{5}{\sqrt{2}}$$

at $t = \frac{1}{2}$,

$$x\left(\frac{1}{2}\right) = \sin\left(1000\frac{\pi}{2}\right) = 0$$

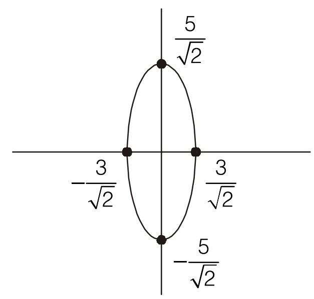
$$y\left(\frac{1}{2}\right) = 5\cos\left(500\pi + \frac{\pi}{4}\right) = \frac{5}{\sqrt{2}}$$

at $t = \dfrac{1}{4000}$,

$$x\left(\frac{1}{4000}\right) = 3\sin\frac{\pi}{4} = \frac{3}{\sqrt{2}}$$

$$y\left(\frac{1}{4000}\right) = 5\cos\left(\frac{\pi}{4}+\frac{\pi}{4}\right) = 0$$

Thus, this represents the equation of ellipse.



1.20 (c)

Let $n = 2$ and $A = \begin{bmatrix} p & q \\ q & r \end{bmatrix}$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

then, $\quad f(x) = X^T AX + B^T X + C$

$$= (px^2 + 2qxy + ry^2) + (b_1x + b_2y) + C$$

$$\frac{\partial f}{\partial x} = 2px + 2qy + b_1 = 0 \qquad \text{..(i)}$$

$$\frac{\partial f}{\partial y} = 2qx + 2ry + b_2 = 0 \qquad \text{...(ii)}$$

From equations (i) and (ii)

$$2\begin{bmatrix} p & q \\ q & r \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = -\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

i.e. $\quad 2AX = -B$

$$\Rightarrow \quad AX = -\frac{B}{2}$$

$$\therefore X = -\frac{A^{-1}B}{2} \quad [\because A \text{ is +ve definitely} \Rightarrow |A| \geq 0]$$

$$r = \frac{\partial^2 f}{\partial x^2} = 2p \;;\; s = \frac{\partial^2 f}{\partial x \partial y} = 2q$$

$$t = \frac{\partial^2 f}{\partial y^2} = 2r$$

$$rt - s^2 = 4(pr - q^2) > 0$$

$$r = 2p > 0 \qquad [\because A \text{ is +ve definitely}]$$

$$\therefore \quad \text{At } X = -\frac{A^{-1}B}{2}$$

We get minimum of $f(x)$.

Similarly we can verify for $n > 2$ also.

1.21 (c)

$$[A] = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \\ 4 & 5 & 6 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Eigen values

$$[A - \lambda I] = \begin{bmatrix} (-1-\lambda) & 0 & 0 \\ 0 & (1-\lambda) & 0 \\ 0 & 0 & (\lambda) \end{bmatrix} = 0$$

$$(-1-\lambda)\left[(1-\lambda)^2\right] = 0$$

$$-1 - \lambda = 0 \qquad\qquad (1-\lambda)^2 = 0$$

$$\boxed{\lambda = -1} \qquad \boxed{\lambda = 1, 1}$$

$$\therefore \qquad (\lambda = -1, 1, 1)$$

1.22 (c)

Given, $\quad AX = 0$

$$\rho(A_{n\times n}) = r\,(0 < r < n)$$

p = Number of independent solutions = nullity

We know that

$$\text{rank} + \text{nullity} = n$$

$$r + p = n$$

$$p = n - r$$

1.23 (3)

Equation of straight line,

$$y = mx + c$$

passing through (0, 0)

$$0 = 0 + c \quad \Rightarrow c = 0$$

$$y = mx$$

Passing through (2, 6)

$$\therefore \quad 6 = 2\,m$$

$$\therefore \quad m = 3$$

1.24 (–6)

Eigen values of given matrix A are 1, –1, 3

Eigen values of A^3 are	1,	–1,	27
Eigen values of $3A^2$ are	3,	3,	27
Eigen values of $A^3 - 3A^2$ are	–2,	–4,	0

trace of $A^3 - 3A^2 = -2 - 4 + 0 = -6$

1.25 (c)

Characteristics equation is $|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & -1 & 5 \\ 0 & 5-\lambda & 6 \\ 0 & -6 & 5-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)((5-\lambda)^2 + 36) + 1(0-0) + 5(0-0) = 0$$

$(1-\lambda)(\lambda^2 - 10\lambda + 61) = 0$

$\lambda = 1,$

$$\lambda = \frac{10 \pm \sqrt{100-244}}{2} = \frac{10 \pm 12i}{2} = 5 \pm 6i$$

$\lambda = 1,\ 5 \pm 6i$

1.26 (d)

Coordinate transformation matrix

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Orthogonal $\therefore$ $\theta = 90°$

Coordinate transformation matrix of mirror image

$$= \begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{bmatrix}$$

$$= \begin{bmatrix} \sin 90° & \cos 90° \\ \cos 90° & -\sin 90° \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

1.27 (1)

Since V is non-zero vector of dimension 3×1

Therefore, $\rho(A) \le \min\{\rho(V), \rho(V^T)\}$

$\le \min\{1, 1\}$

≤ 1

Since V is non-zero. Hence $\rho(A) = 1$

1.28 (a)

$N^2 = 0$

Let Eigen values of N are $\lambda_1, \lambda_2, \lambda_3$;

Eigen values of N^2 are $\lambda^2{}_1, \lambda^2{}_2, \lambda^2{}_3$

But $N^2 = 0$

$\Rightarrow$ $\lambda_1^2 = 0,\ \lambda_2^2 = 0,\ \lambda_3^2 = 0$

$\therefore$ Eigen values of N are 0, 0, 0.

1.29 (1)

$(x-2)^2 = 2x - 1$

$x^2 - 6x + 5 = 0$

$x = 1, 5$

Smallest value of x is 1.

1.30 (c)

Put $k = -3$ in options,

$3x + 2ky = -2 \Rightarrow 3x - 6y = -2$

$kx + 6y = 2 \Rightarrow 3x - 6y = -2$

Both the equations are reducing in the same form, thus the system will have infinite number of solutions.

1.31 (10)

Product of eigen values $= |A|$

$= (1)(2)(5) = 10$

1.32 (b)

$$y = mx + c = \frac{dy}{dx}x + c$$

(given that Y intercept is $-y$)

$$y = \frac{dy}{dx}x - y$$

$$\partial y = \frac{dy}{dx}x$$

$$\frac{dy}{\partial y} = \frac{dx}{x}$$

$\Rightarrow$ $\frac{1}{2}\ln y = \ln x + \ln C$

$\ln y^{1/2} = \ln(xC)$

$y = x^2C^2$

$\therefore$ $y \propto x^2$

1.33 (d)

Matrix A has rank 2

Matrix A^T has rank 2

$\Rightarrow AA^T$ has rank 2

$\therefore$ $|AA^T{}_{4\times 4}| = 0$

Hence system cannot have unique solution.

1.34 (b)

$\because$ $C_1 + C_2 + C_3 = 0 \Rightarrow \lambda = 0$

We know that, $MX = \lambda X \Rightarrow MX = 0$

$$\begin{bmatrix} 1 & -1 & 0 \\ 1 & -2 & 1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Consider two equations

$x - y - 0 \cdot z = 0$

$x - 2y + z = 0$

Solving $\frac{x}{-1} = \frac{y}{-1} = \frac{z}{-1} = K$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = K_1\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Hence one of the eigen vector is $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

1.35 (4)

Characteristic Eq. of A is $|A - \lambda I| = 0$

$\Rightarrow \quad (\lambda - 2)(\lambda - 3) = 0 \quad \Rightarrow \quad \lambda^2 - 5\lambda + 6 = 0$

By C.H theorem replace $\lambda \to A$ in characteristic equation

$$A^2 - 5A + 6I = 0 \qquad \ldots(1)$$

Now,

$|A^4 - 5A^3 + 6A^2 + 2I| = |A^2(A^2 - 5A + 6I) + 2I|$

$= |0 + 2I| = |2I| = 2^2|I| = 4|I| = 4 \times 1 = 4$

1.36 (d)

$$A = 2\begin{bmatrix}1\\0\end{bmatrix}[1 \;\; 0] + 3\begin{bmatrix}2\\0\end{bmatrix}[2 \;\; 0]$$

$$= 2\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} + 3\begin{bmatrix}4 & 0\\0 & 0\end{bmatrix}$$

$$= \begin{bmatrix}2 & 0\\0 & 0\end{bmatrix} + \begin{bmatrix}12 & 0\\0 & 0\end{bmatrix} = \begin{bmatrix}14 & 0\\0 & 0\end{bmatrix}$$

$\rho(A) = 1$

1.37 (4)

Expanding along R_1 (Row 1)

$$|A| = +(1)\begin{vmatrix}4 & 0 & 0\\0 & 4 & 3\\0 & 2 & 1\end{vmatrix} - (2)\begin{vmatrix}3 & 0 & 0\\0 & 4 & 3\\0 & 2 & 1\end{vmatrix}$$

$$= (1)[4(4 - 6)] - 2[3(4 - 6)] = 4$$

1.38 (a, d)

Given matrix, $$M = \begin{bmatrix}2 & 3 & 7\\6 & 4 & 7\\4 & 6 & 14\end{bmatrix}$$

Applying $R_3 \to R_3 - 2R_1$

$$M = \begin{bmatrix}2 & 3 & 7\\6 & 4 & 7\\0 & 0 & 0\end{bmatrix}$$

Rank of matrix is 2.

Rank of matrix is less than 3.

Hence, determinant is zero.

1.39 (a, c)

Eigen values given,

$$\lambda = -5, 7$$

Now for eigen values,

$$\lambda_1 = -5$$

$$[A - \lambda_1 I][x] = 0$$

$$\begin{bmatrix}4-(-5) & 3\\9 & -2-(-5)\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix}9 & 3\\9 & 3\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$9x_1 + 3x_2 = 0$$

$$x_2 = -3x_1$$

Let $x_1 = K$, then $x_2 = -3K$

So, eigen vector for $\lambda_1 = 5$,

$$= K\begin{bmatrix}1\\-3\end{bmatrix}$$

For eigen value $\lambda_2 = 7$

$$\begin{bmatrix}4-7 & 3\\9 & -2-7\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix}-3 & 3\\9 & -9\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

$$-3x_1 + 3x_2 = 0$$

$$x_1 = x_2$$

So, eigen vector is $K\begin{bmatrix}1\\1\end{bmatrix}$.

1.40 (a)

Given : $x - 2y + z = 0$

$x - z = 0$

Solution is given by solving also equations.

	x		y		z	
–2		1		1		–2
0		–1		1		0

$$\frac{x}{2} = \frac{y}{2} = \frac{z}{2} = K$$

$\Rightarrow \quad x = K,\; y = K,\; z = K$

$$\therefore \quad \begin{bmatrix}x\\y\\z\end{bmatrix} = K\begin{bmatrix}1\\1\\1\end{bmatrix},\; K \in R$$

1.41 (–8)

Given : $\rho(A_{3\times 2}) = 1$

So, $\quad C_2 = KC_1$

$\Rightarrow \quad 4 = K(1)$

$\Rightarrow \quad K = 4$

Also, $\quad \alpha = -3K$ and $6 = K\beta$

$$\therefore \quad \frac{\alpha}{\beta} = -\frac{1}{2}K^2 = -\frac{1}{2}\times 4^2 = -8$$

■■■